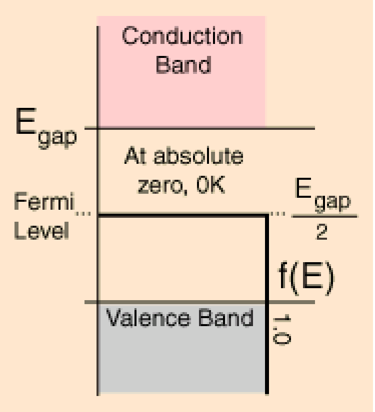
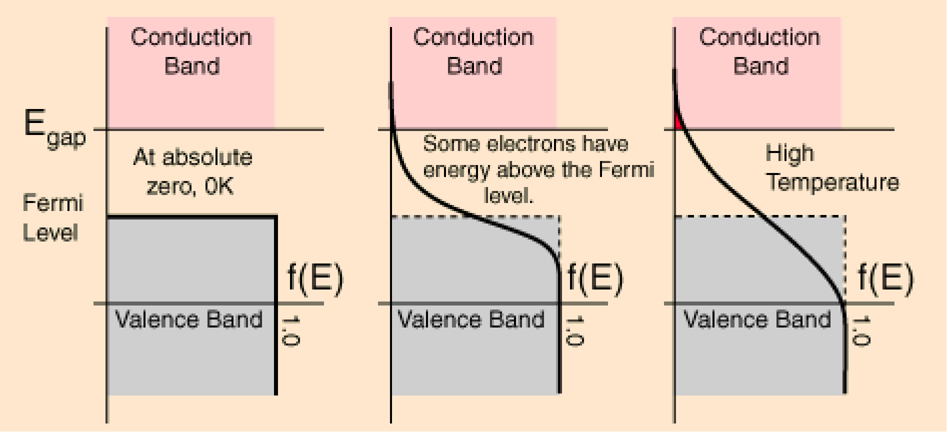
Fermi Function

Probability that a given available electron energy state will be occupied at a given temperature.

$$f(E) = \frac{1}{e^{(E - E_F)/kT} + 1}$$



Context of Fermi level for a semiconductor



"Fermi level" is the term used to describe the top of the collection of electron energy levels at absolute zero temperature. This concept comes from Fermi-Dirac statistics. Electrons are fermions and by the Pauli exclusion principle cannot exist in identical energy states. So at absolute zero they pack into the lowest available energy states and build up a "Fermi sea" of electron energy states. The Fermi level is the surface of that sea at absolute zero where no electrons will have enough energy to rise above the surface. The concept of the Fermi energy is a crucially important concept for the understanding of the electrical and thermal properties of solids. Both ordinary electrical and thermal processes involve energies of a small fraction of an electron volt. But the Fermi energies of metals are on the order of electron volts. This implies that the vast majority of the electrons cannot receive energy from those processes because there are no available energy states for them to go to within a fraction of an electron volt of their present energy. Limited to a tiny depth of energy, these interactions are limited to "ripples on the Fermi sea".

At higher temperatures a certain fraction, characterized by the Fermi function, will exist above the Fermi level. The Fermi level plays an important role in the band theory of solids. In doped semiconductors, p-type and n-type, the Fermi level is shifted by the impurities, illustrated by their band gaps. The Fermi level is referred to as the electron chemical potential in other contexts.

The Fermi energy also plays an important role in understanding the mystery of why electrons do not contribute significantly to the specific heat of solids at ordinary temperatures, while they are dominant contributors to thermal conductivity and electrical conductivity. Since only a tiny fraction of the electrons in a metal are within the thermal energy kT of the Fermi energy, they are "frozen out" of the heat capacity by the Pauli principle.

The Fermi function f(E) gives the probability that a given available electron energy state will be occupied at a given temperature.
The Fermi function comes from Fermi-Dirac statistics and has the form

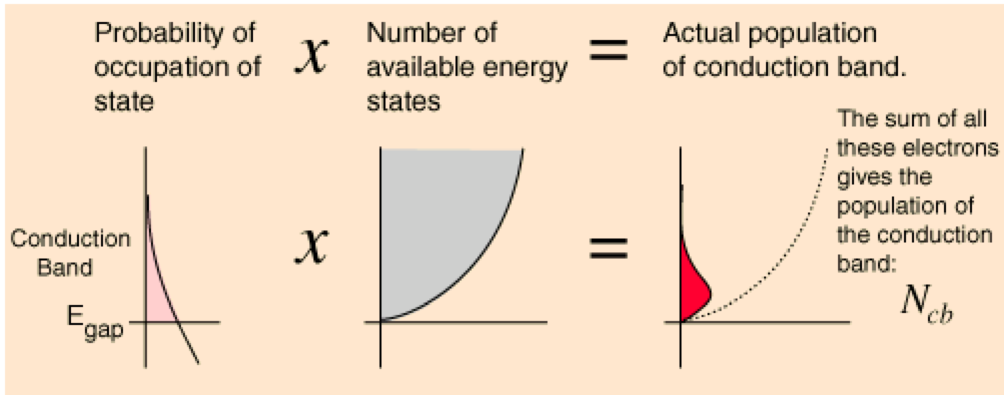
$$f(E) = \frac{1}{e^{(E - E_F)/kT} + 1}$$

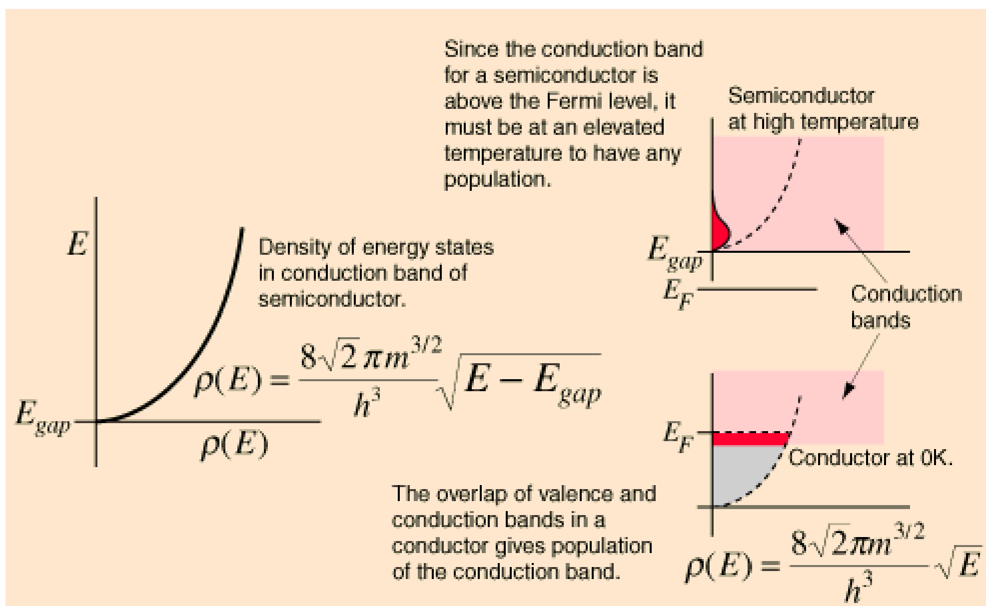
The basic nature of this function dictates that at ordinary temperatures, most of the levels up to the Fermi level EF are filled, and relatively few electrons have energies above the Fermi level. The Fermi level is on the order of electron volts (e.g., 7 eV for copper), whereas the thermal energy kT is only about 0.026 eV at 300K. If you put those numbers into the Fermi function at ordinary temperatures, you find that its value is essentially 1 up to the Fermil level, and rapidly approaches zero above it.

The illustration above shows the implications of the Fermi function for the electrical conductivity of a semiconductor. The band theory of solids gives the picture that there is a sizable gap between the Fermi level and the conduction band of the semiconductor. At higher temperatures, a larger fraction of the electrons can bridge this gap and participate in electrical conduction.

Note that although the Fermi function has a finite value in the gap, there is no electron population at those energies (that's what you mean by a gap). The population depends upon the product of the Fermi function and the electron density of states. So in the gap there are no electrons because the density of states is zero. In the conduction band at 0K, there are no electrons even though there are plenty of available states, but the Fermi function is zero. At high temperatures, both the density of states and the Fermi function have finite values in the conduction band, so there is a finite conducting population.

Density of states, Fermi function, Conduction band





The Fermi function gives the probability of occupying an available energy state, but this must be factored by the number of available energy states to determine how many electrons would reach the conduction band.This density of states is the electron density of states, but there are differences in its implications for conductors and semiconductors. For the conductor, the density of states can be considered to start at the bottom of the valence band and fill up to the Fermi level, but since the conduction band and valence band overlap, the Fermi level is in the conduction band so there are plenty of electrons available for conduction. In the case of the semiconductor, the density of states is of the same form, but the density of states for conduction electrons begins at the top of the gap.